Hard jet probes in terms of colorless QCD dipoles

R. Peschanski*

The forward jet playing the rôle of a hard probe in high energy QCD scattering is analyzed in terms of QCD dipole (color-singlet $q\bar{q}$) configurations in the transverse position space. There are sizable differences with the analoguous $q\bar{q}$ configurations of a hard photon probe which may lead to significant phenomenological consequences on the analysis of hard processes using a forward jet probe, e.g. at the Tevatron. A geometrical interpretation of the resulting distribution in terms of black disk diffraction is proposed.

1. Studies on deep-inelastic scattering experiments by virtual photon probes have shown the interest of using the formalism of $q\bar{q}$ wave-functions of the photon in the infinite momentum frame [1]. Indeed, the theoretical predictions for small-x structure functions at the leading-log approximation and in the multicolor $1/N_c$ limit can be expressed in terms of the probability of finding the $q\bar{q}$ configurations inside the photon probe convoluted with the high-energy dipole-dipole cross-section [2–4]. This scheme provides a useful description of the hard probe in the *configuration space* of the variables $\left(\vec{r} \equiv re^{i\phi}, z \right)$ where r is the transverse size of the $q\bar{q}$ state, φ, the azimuthal orientation over the photon momentum and z (resp. $1-z$) is the energy fraction brought by the quark (resp. the antiquark) in the infinite momentum frame.

This scheme can be proved equivalent [5] to the k-factorization property [6] of structure functions calculated at small x_{Bj} : a QCD photon vertex function is obtained convoluted with the unintegrated off-shell gluon structure function solution of the Balitskii, Fadin, Kuraev, Lipatov (BFKL) equation [7].

For high-energy QCD processes in general, the use of photon wave-functions are very useful for the investigations on the so-called "hard diffraction" processes characterized by a large rapidity gap between particles produced at the photon and proton vertices at HERA. Indeed, the physical process by which a color singlet $q\bar{q}$ configuration of the photon propagates in the configuration space and interacts with the proton gives an interesting insight on the dynamics of "hard diffraction" initiated by a photon probe [8].

In other reactions of interest, the hard probe is not furnished by a virtual photon. This is the case when an energetic forward jet with high transverse momentum k_T serves as the hard QCD probe in various studies such as the Mueller-Navelet process [9] and also "hard diffraction" processes under study at the Tevatron by various combinations of jets and gaps (in the following we consider only one hard jet, independently of the way the transverse momentum is balanced, e.g. by another jet). The forward jet as a hard probe is already present in HERA studies where it allows a comparison with perturbative QCD predictions for the hard-gluon on hard-photon scattering at high energy [10]. However, if a treatment of the forward jet or gluon vertex following the original BFKL scheme is known [11], the corresponding formalism using the propagation of $q\bar{q}$ color-singlets and their interaction with the target has not been yet discussed, to our knowledge. Our goal is to derive an estimate of the color-singlet dipole content of a forward jet probe and to discuss its configuration space characteristics by comparison with the photon case.

It is worthwhile to emphasize the similarities and differences expected between the singlet configurations of photon and gluon probes. In the photon case (see Fig.1a) the color singlet $q\bar{q}$ configurations are given by the light-cone probability distributions Φ^{γ} and interact *via* the dipole-dipole cross-section σ_d with the *primordial* dipole distribution in the target Φ^t.

In the case of a forward gluon jet we want to consider, the situation is more delicate (see Fig.1b). The forward jet production is mediated by a virtual gluon (or a sea quark, which is equivalent in the multicolor $1/N_c$ limit of QCD we consider) which carries color and thus is not directly analyzable in terms of color singlet $q\bar{q}$ configurations; It is more likely a three-body than a two-body problem since it involves at least a soft exchanged gluon which carries away the color (see the Fig.1b). However, it is possible to make a reasonable assumption allowing to treat the problem in a way similar to the photon. Indeed, the BFKL interaction initiated by an initial (virtual) gluon can as well be described in terms of dipole-dipole interactions, e.g. in terms of color singlet configurations. Singularizing the most energetic dipole (which we identify as the *primordial* dipole in the forward jet formation), we can define its light-cone probability distribution Φ^g, interacting through the same dipole-dipole cross-section σ_d and the same vertex dipole distribution Φ^t as in the photon case. As we shall see, within the approximation that the abovementionned exchanged gluon carries little energy fraction to the remaining interacting dipoles, this QCD dipole factorization properties lead to a prediction for Φ^g, where the gluon is specified to be with transverse polarization only.

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*CEA, Service de Physique Theorique, CE-Saclay, F-91191 Gif-sur-Yvette Cedex, France

The plan of our study is the following. In the next section **2**, we briefly introduce the appropriate QCD dipole formalism and derive the resulting prediction for the integrated Mellin transformed $\tilde{\Phi}^g$ of the distribution Φ^g we look for. Using a physically motivated assumption on the overall energy carried by the *primordial* dipole, the last step of the derivation of Φ^g is made in section **3**, with a discussion on the properties of our result. Conclusion and questions for future work appear in section **4.**

2. For a given target t, the transverse projectile-target (where the projectile is either a photon or a gluon) total cross sections σ^t obtained by high-energy QCD in the leading logs would read in the dipole scheme

$$\sigma^t = \int d^2 r dz \; \Phi(r, z; Q^2) \int d^2 r_t dz_t \; \Phi^t(r_t, z_t; Q_t^2) \; \times \sigma_d(r, r_t; Y) \; , \tag{1}$$

where we introduced the probability distributions $\Phi(r, z; Q^2)$ for the transversely polarized projectile and $\Phi^t(r_t, z_t; Q_t^2)$ for the primordial dipole distribution inside the target. The dipole-dipole cross section $\sigma_d(r, r_t; Y)$ within the (large) rapidity range Y can be expressed [5] using an inverse Mellin transform:

$$\sigma_d(r, r_t; Y) = 2\pi\alpha_s^2 \int \frac{d\gamma}{2i\pi} \frac{\left(r^2\right)^{1-\gamma}\left(r_t^2\right)^\gamma}{\gamma^2(1-\gamma)^2} \; e^{\epsilon(\gamma)Y} \; , \tag{2}$$

where

$$\epsilon(\gamma) = \frac{\alpha_s N_c}{\pi} \; [2\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma)] \tag{3}$$

is the (Mellin transformed) BFKL kernel [7] with $\Psi(\gamma) \equiv d\log\Gamma/d\gamma$.

Let us come to our derivation. Since in formula (1) the target distribution $\Phi^t(r_t, z_t; Q_t^2)$ can be expanded over individual massive dipole components $\Phi^t \equiv 1/\pi \; \delta(r_t^2 - 1/Q_t^2)\delta(z_t - 1/2)$, we shall restrict ourselves to the gluon-dipole cross-section writing

$$\sigma = \frac{4\pi\alpha_s^2}{Q^2} \int \frac{d\gamma}{2i\pi} \left(\frac{Q}{Q_t}\right)^{2\gamma} e^{\epsilon(\gamma)Y} \frac{\tilde{\phi}^g(\gamma)}{\gamma^2(1-\gamma)^2} \; , \tag{4}$$

where

$$\int d^2 r \left(r^2 Q^2\right)^{1-\gamma} \int dz \; \Phi^g(r, z; Q^2) = \tilde{\phi}^g(\gamma) \tag{5}$$

is the integrated (over the energy fraction z) and mellin-transformed (over the transverse coordinate space) of the *primordial* dipole distribution out of an incident virtual gluon.

The BFKL solution for the same gluon-dipole cross-section writes [4,5]

$$\sigma = \frac{\pi^2\alpha_s^2}{2Q^2} \int \frac{d\gamma}{2i\pi} \left(\frac{Q}{Q_t}\right)^{2\gamma} e^{\epsilon(\gamma)Y} \frac{v(\gamma)}{(1-\gamma)} \; . \tag{6}$$

It is obtained using the *unintegrated* k-factorized gluon coupling to a $q\bar{q}$ pair which is given[1] (for the lower vertex) by

$$v(\gamma) = 2^{-2\gamma-1} \frac{\Gamma(1-\gamma)}{\gamma\Gamma(1+\gamma)} \; . \tag{7}$$

Note the factor $(1-\gamma)^{-1}$ in the integral (6) due to the expression of cross-sections usually given after integration over k_T above a certain value Q which plays the rôle of the virtuality (see, e.g. [5]).

It is important to notice a subtlety about the incoming gluon which initiates the process. The incident gluon acquires a different status than the exchanged ones appearing in the BFKL derivation of σ_d. Indeed, it is known that the BFKL gluons are reggeized and predominantly longitudinally polarized in the axial gauge [3]. On the other hand the incident gluon is predominantly transversely polarized in the same gauge [12] since it comes from the DGLAP evolution from

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[1]These *unintegrated* couplings can be deduced from the factorization of the elementary dipole-dipole cross-section (2) since $v(\gamma)v(1-\gamma) \equiv 1/16\gamma^2(1-\gamma)^2$, where $v(\gamma)$ corresponding to formula (7) is for the lower vertex and $v(1-\gamma)$ is for the upper one.

the physical projectile (e.g. a proton). This non trivial helicity-flip is not contradictory to the properties of the BFKL Pomeron (which contains the DGLAP evolution at the double leading log accuracy) since it can be seen from the high-energy properties of the gluon propagators and polarization vectors in the BFKL derivation [13]. From that property comes the fact that we single out our prediction for the transversely polarized gluon only.

By consistency of both equivalent formulations (4,6), in the same way as it has been done for the photon case [5], one gets the following result

$$\tilde{\phi}^g(\gamma) = 8\gamma^2(1-\gamma)\ v(\gamma) \equiv 2^{2-2\gamma}\frac{\Gamma(2-\gamma)}{\Gamma(\gamma)}\ . \tag{8}$$

This expression is to be compared with the analogous one for the photon namely [4,5]:

$$\tilde{\phi}^\gamma = C\frac{1}{\gamma(1-\gamma)}\ \frac{\Gamma^3(1+\gamma)\Gamma(2+\gamma)\Gamma(2-\gamma)\Gamma(3-\gamma)}{\Gamma(2+2\gamma)\Gamma(4-2\gamma)}\ , \tag{9}$$

where the normalization factor ($C = \alpha_{em}N_c e_q^2/2\pi^2$ with e_q the quark charge) takes into account in this case the color, flavor and electromagnetic charges.

3. The solution for the photon case, corresponding to the inversion of the integrated and Mellin transformed function $\tilde{\phi}^\gamma$ of formula (9), is well known [1,2]. The probability distribution of the light-cone color-singlet $q\bar{q}$ configurations for a transversely polarized virtual photon writes

$$\Phi^\gamma\left(z, r, Q^2\right) \equiv C\left(z^2 + (1-z)^2\right)\ \hat{Q}^2\ \tilde{\phi}^\gamma(u)\ ;\ \tilde{\phi}^\gamma(u) = K_1^2(u), \tag{10}$$

where K_1 is the Bessel function of second kind and by definition $u \equiv r\hat{Q} \equiv rQ\sqrt{z(1-z)}$. The z-dependent prefactors are known [1] to come from the coupling of the quark and antiquark spinors to the transverse helicities of the virtual photon. Our task is now to find a similar inversion procedure for formula (5), i.e. applicable to the gluon distribution.

To our knowledge, the z-dependence of the $q\bar{q}$ system of the *primordial* dipole from a gluon probe is not known. We shall make use in the subsequent derivation of a plausible assumption (at least for z not too near $(0,1)$, namely that the *primordial* dipole in the gluon jet retains the most part of the energy and longitudinal momentum of the initial gluon. This assumption can be based on the physical idea that the color degrees of freedom are much more "volatile" than the energy momentum[2], which means that the energy balance in a local interaction $g \to (colored)q\bar{q}$ is not changed when transmuted to the system $g \to (soft)g + (singlet)q\bar{q}$ as schematized in Fig.1b. In other terms we assume a hierarchical structure of the cascade of interacting dipoles in the rapidity space which keeps the most part of energy in the first rank in rapidity, i.e. the *primordial* dipole. In practical terms, the exchanged colored gluon (see the upper vertex in Fig.1b) brings a negligeable energy fraction out of the quark (or antiquark) to which it is coupled and thus the balance in the infinite momentum frame between the energy fraction z brought by the quark and $1-z$ for the antiquark remains approximatively the same as for the direct coupling of the gluon to $q\bar{q}$.

Thus, in this framework, we are led to the same kinematical prefactors for the gluon as for the photon (10), namely we write

$$\Phi^g(r, z; Q^2) = \left(z^2 + (1-z)^2\right)\ \hat{Q}^2\ \phi^g(u) \tag{11}$$

With this ansatz, the solution boils down to solve an inverse Mellin transform which gives:

$$\phi^g(u) = \frac{1}{\pi}\int\frac{d\gamma}{2i\pi}u^{2(\gamma-2)}\ 2^{1-2\gamma}\frac{\Gamma(2-\gamma)\Gamma(2\gamma+2)}{\Gamma(\gamma+2)\Gamma^2(\gamma)} = \frac{5}{2\pi}\ {}_2F_3\left(\frac{7}{2}, 3; 4, 2, 2, -u^2\right)$$

$$\equiv \frac{1}{\pi}\left\{\frac{J_1^2(u)}{u^2} + J_0^2(u) - J_1^2(u) - \frac{J_2^2(u)}{2u^2}\right\}\ , \tag{12}$$

where $J_{0,1,2}$ are the usual Bessel functions of first kind. The normalization is fixed in such a way that $\int d^2rdz\ \Phi^g(r, z; Q^2) \equiv 1$. the hypergeometric function ${}_2F_3$ has thus been finally cast [15] into a form similar to

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[2]The same hypothesis has led to interesting models for "hard diffraction" connecting color-singlet production to the kinematics of perturbative vertices [14].

the photon case, but with a combination of Bessel functions of first kind instead of the squared of the Bessel function of second kind appearing for the photon in (10).

In the following discussion we shall comment on the interesting differences with the photon case (10) which appear clearly on Fig.2, where both functions $\tilde{\phi}$ have been drawn.

i) Fig.2 shows the function ϕ^g compared to ϕ^γ as a function of $u^2 = Q^2 z(1-z)$ in the region $u^2 \leq 2$. The existence of regions with negative ϕ^g beyond this $u^2 = 2$ shows that a simple interpretation in terms of probability distributions -as for the photon case [1]- cannot be valid. We shall dicuss later a possible interpretation of this fact.

ii) Fig.2 also reveals that the space-time configuration of the *primordial* singlet state resulting from the virtual gluon is likely to be much more spread in configuration space than in the photon case. Indeed, the combination of Bessel functions in formula (12) has a tail behaving as $\cos(2u)/u$, while the photon distribution tail is exponentially small as $K_1^2(u) \sim \exp(-2u)/u$. The difference is already visible at finite values of u 62 as shown by Fig.2. As a consequence, for a given scale $\hat{Q}$ of the hard probe, there can be larger fluctuations in the dipole size than for the photon. Thus, even a reasonably hard scale probe can give rise to large dipoles for which, e.g., saturation and/or screening corrections may be more important than for the photon. This remark may be phenomenologically relevant when comparing hard QCD processes at high energy at the Tevatron and at Hera.

iii) The formula (12) with both positive and negative contributions may have an interesting interpretation in terms of real and virtual contributions of simple geometrical meaning. Indeed, it is well known that the dipole wave-function formalism [3] implies both terms. It is thus conceivable that factorizing out the contribution of the *primordial* dipole keeps this feature. Note that the virtual contributions are necessary to obtain an overall equivalence between the dipole model and the BFKL formalism, even if, separately, real and virtual contributions are different in both approaches [16].

iv) A nice geometrical interpretation of our resulting formula (12) comes from the comparison with the well-known amplitudes for diffraction by a black disk [17]. In a high-energy version taking into account also the helicity structure [18] the black disk amplitudes write

$$M_n^{el}(\hat{Q}) = \hat{Q}^2 \int_0^\infty b^n db^2 J_n(b\hat{Q}) \ \theta(b^2 - r^2) \propto \frac{J_{n+1}(r\hat{Q})}{r\hat{Q}} \ , \tag{13}$$

for elastic amplitudes on the bulk of the black disk and

$$M_n^{inel}(\hat{Q}) = \hat{Q}^2 \int_0^\infty b^n db^2 J_n(b\hat{Q}) \ \delta(b^2 - r^2) \propto J_n(r\hat{Q}) \ , \tag{14}$$

for inelastic amplitudes corresponding to peripheral reactions on the edge of the disk. n is the net helicity flip in the reaction [18].

If we are allowed to interpret the Bessel functions appearing in (12) as wave-function components (both for real and virtual contributions), a geometrical interpretation would be that the *primordial* dipole totally *absorbs* the partial waves for impact parameter $b \leq r$, where r is its size.

4. In conclusion, assuming the quasi-two-body kinematical coupling of an initial virtual gluon to the *primordial dipole* of cascading dipoles describing the BFKL amplitudes, we have derived the analogue of the transverse photon probability distribution (10) for a transverse gluon, see formulae (12) . The solution differs from the photon, in particular by its wider extension in configuration space, allowing for larger fluctuations of the dipole sizes for a given gluon virtuality. An interpretation in terms of black disk absorption has been proposed.

Some questions arise from our analysis.

-Can we discuss the existence, properties and limitations of a configuration space distribution ϕ^g *directly* from the three-body problem [19] $g \to (soft)g + singlet(q\bar{q})$, corresponding to the coupling of the initial gluon to the *primordial* dipole and another soft gluon bringing out the color quantum numbers, see Fig.1?

-Are the real/virtual states decomposition suggested by our resulting formula valid and does it corresponds (in the QCD dipole basis) to the existence of *primordial* colored states in the gluon wave-function?

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-Can we say something about the longitudinal polarization of the gluon by comparison with the known result for the photon[3] ?

We think worthwhile to address these questions, since the phenomenology of forward jets as QCD hard probes, useful e.g. at the Tevatron, requires a better understanding of the virtual gluon in configuration space. This could shed a new light (or give new tools) to high energy and diffractive QCD scattering at hadron colliders and perhaps help understanding the physical origin of non-factorization or saturation mechanisms for jet-induced hard QCD scattering.

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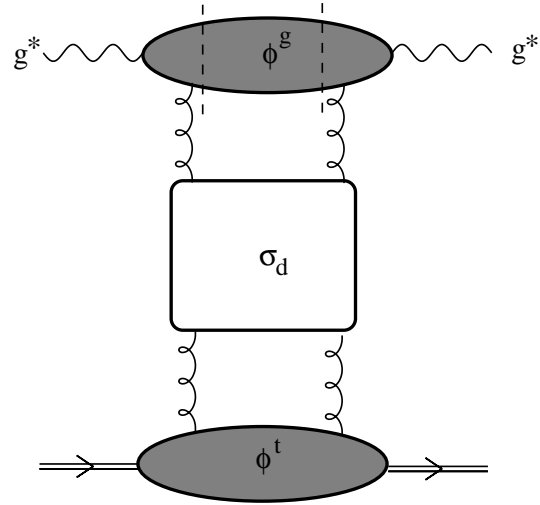[19] For an interesting approach see S.Munier, to appear.

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[3] We remark that assuming by example the same dipole coupling (4) for both polarizations of the gluon leads also to a simple decomposition in terms of bessel functions, namely

$$\Phi_L^g\left(z, r, Q^2\right) \equiv \frac{3}{\pi} z(1-z) \ \hat{Q}^2 \ \left\{ \frac{J_1^2(u)}{u^2} + J_0^2(u) - \frac{2}{3} J_1^2(u) - \frac{1}{3} J_2^2(u) \right\} \ .$$
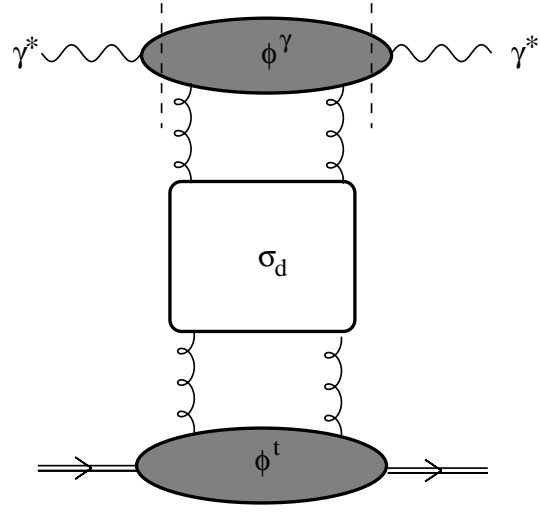
Figure 1

Photon and gluon-initiated reactions in the QCD dipole basis

The figures schematically describe the QCD dipole contributions to photon and gluon-induced reactions. Fig 1a: the photon-induced reaction; Fig 1b: the gluon-induced reaction. The photon (resp. gluon) probability distribution is denoted by ϕ^γ (resp. ϕ^g). ϕ^t is the target dipole distribution and σ_d the dipole-dipole cross-section, see the corresponding formulae in the text.

In both cases the intermediate singlet $q\bar{q}$ state discussed in the text is marked by a dotted line.
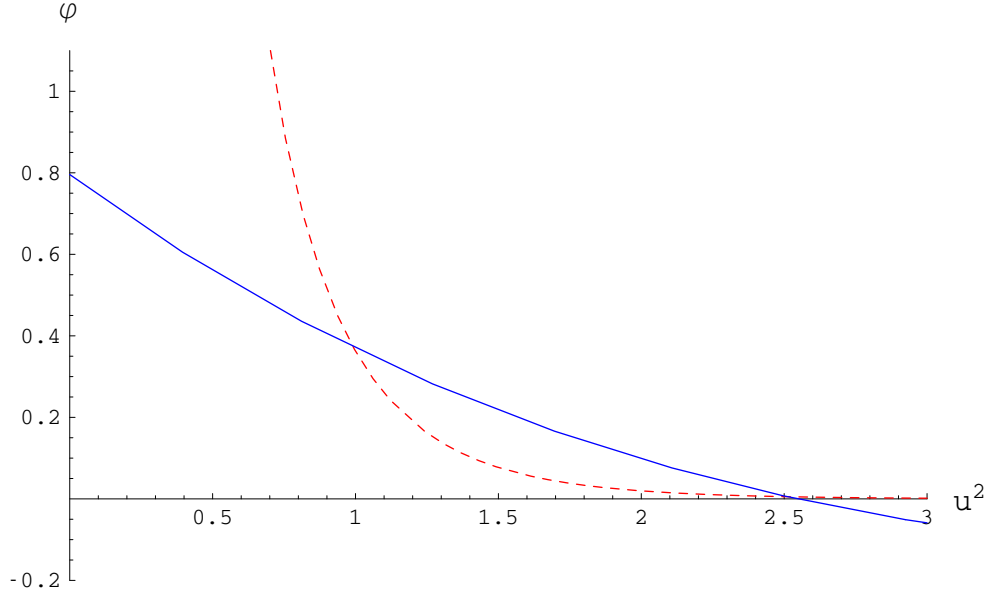
Figure 2

The gluon probe vs. the photon probe

The figure represents the gluon transverse polarization distribution ϕ^g, formula (12), compared to ϕ^γ, formula (10), for the photon (with the constant C equal to 1 for making easier the comparison). The abcissa is defined by $u^2 \equiv r^2 Q^2 z(1-z)$.

Continuous line: the gluon probe distribution; Dashed line: the photon probe distributions.